Week 4 Pre-Lab Reading

1 Introduction to Interferometry

Telescopes have two primary purposes: to collect as much light as possible, and to resolve objects that are close together. Telescope with larger mirrors or dishes have more collecting area and therefore gather more light, which makes then more sensitive to faint sources. A large telescope also has better resolution due to the larger mirror or dish. The angular resolution θ (measured in radians) of a single telescope is given by:

$$\theta = 1.22\frac{\lambda}{D} \tag{1}$$

where λ is the wavelength of light you're observing with, and D is the diameter of the mirror or dish on the telescope.

The angular resolution θ represents the angle at which two objects can be distinguished from each other, so smaller values of θ represent a better resolution. Increasing the size of a telescope gives better resolution, however there's a limit on how big telescopes can be made. The largest single mirror optical telescopes in the world are about 8 meters in diameter, trying to build a single mirror larger than this is an extreme engineering challenge. However, the reflective dishes used in a radio telescope don't need to be a perfect mirror finish, and so radio telescope can be made much lighter and larger. For example, the largest radio observatory in the world is the FAST telescope, with a diameter of 500 meters!

However, the size of the mirror or dish is not the only factor that determines the resolution of a telescope. As the wavelength of light (λ) you're observing with becomes larger, the resolution becomes worse. Because the wavelength of radio waves are about 100,000 times longer than optical light, radio telescope have very low resolution compared to optical telescopes.

Radio astronomers found a way around this problem by using a technique called interferometry. The radio waves from multiple telescopes can be brought together and interfered with each other so as to create a virtual telescope that is much larger than the individual telescopes. When using interferometry, the resolution equation becomes:

$$\theta = 1.22\frac{\lambda}{B} \tag{2}$$

where B is the baseline, the separation between the radio telescopes. This baseline can be made arbitrarily large without the need to engineer larger telescopes. For example, the Event Horizon Telescope (EHT) used a network of radio telescopes around the world to create a virtual telescope the size of Earth. The EHT produced an image of the event horizon of a black hole, with the highest resolution ever achieved in astronomy. Using many radio telescopes like this also gives higher sensitivity, as the signal strength from each telescope is added together.

Instead of relying on a single large dish, radio astronomers often use arrays of smaller telescopes spread over a wide area. These telescopes are often identical in design and are strategically positioned to maximize the effectiveness of interferometry. Each telescope in the array collects radio waves from an astronomical source. These signals carry information about the intensity and frequency of the radiation emitted by the objects being observed.

One critical aspect of interferometry is ensuring that the signals received by each telescope are time-synchronized. Precise timing is essential to accurately combine the signals later in the process. This is usually done by using atomic clocks or other highly accurate time-keeping devices. The signals collected by each telescope are then digitized and processed. This involves converting the analog radio waves into digital data using an analog to digital converter (ADC) that can be manipulated and analyzed by computers.

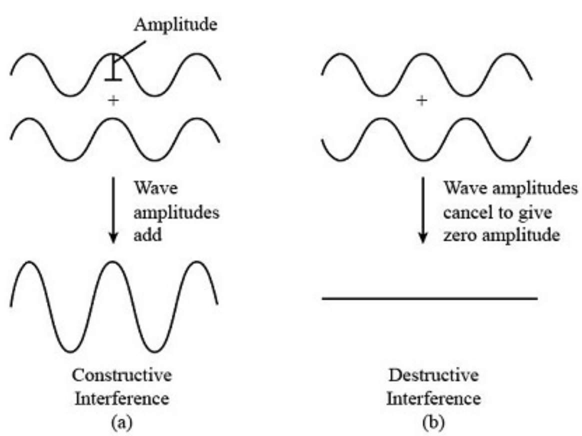

Figure 1: Waves add constructively when in phase, and destructively when out of phase.

Since the telescopes in the array are separated by significant distances, the signals they receive from a given source arrive at slightly different times. This delay is calculated and compensated for during processing. Once the signals are time-aligned, they are combined to constructively interfere in processes known as beamforming and correlation. These processes effectively combines the signals, taking into account their relative phases and intensities. Beamforming creates a focused "beam" of maximum sensitivity towards a specific direction in the sky, which can be steered by adjusting the delays in the signal path, without moving the telescopes themselves. The correlated signals are used to reconstruct an image of the celestial object being observed. This is done using a fast Fourier transform (FFT), which converts the complex interference patterns into meaningful images.

Below is the workflow to get from the initial detection of radio signals at the antennas to a final radio image.

1. **Signal Reception:** The interferometer collects signals from astronomical sources using its antennas.

2. **Beamforming:** The beamformer combines signals from multiple antennas to create a focused beam towards the observed source.

3. **Correlation:** The correlator computes the cross-correlation function between pairs of antennas, measuring the similarity of signals.

4. **Visibility Function:** FFT processors transform the cross-correlation function into the visibility function, which contains spatial frequency information.

5. **Sky Brightness:** Using inverse FFT computations, the sky brightness distribution is reconstructed from the visibility function.

6. **Data Analysis:** DPUs and computing clusters process the reconstructed image data, analyze it, and extract scientific information about celestial objects.

2 Beamforming

Beamforming is a technique used to combine signals from multiple antennas in an array to create a focused "beam" towards a specific direction in the sky. This is achieved by combining elements in the array so that signals at particular angles experience constructive interference and while others experience destructive interference. This process enhances the sensitivity, signal-to-noise ratio (SNR), and angular resolution of a radio telescope, allowing astronomers to detect and study astronomical sources with greater precision and clarity. Beamforming helps in boosting the signal from the target direction while reducing noise from other directions. This improves the SNR, making it easier to distinguish real astronomical signals from background noise.

One of the key benefits of beamforming is its ability to improve spatial resolution. By

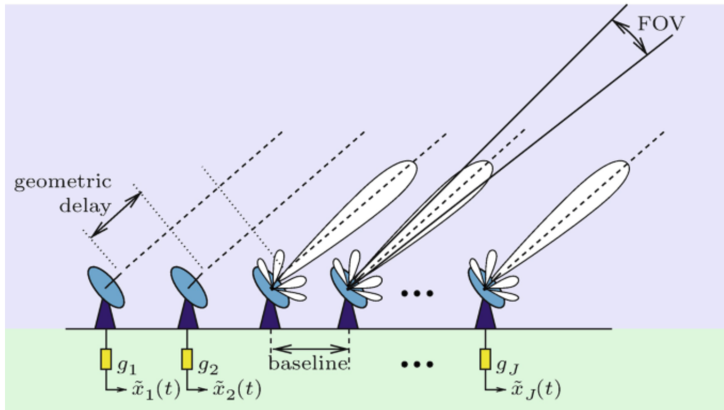

Figure 2: Array of radio telescopes showcasing the beamforming process.

forming a focused beam towards a specific direction, beamforming effectively sharpens the telescope's ability to resolve fine details and structures in celestial objects. Beamforming allows radio telescopes to electronically steer their pointing direction without physically moving the antennas. This flexibility is crucial for observing multiple targets or tracking moving objects in the sky. Beamforming can also help in mitigating interference from terrestrial sources or radio frequency interference (RFI). Beamforming improves the quality of astronomical data by focusing on the desired astronomical signals, and suppressing unwanted signals from other directions.

Consider an array of N antennas, each receiving a signal $x_n(t)$, given by:

$$x_n(t) = s(t) + N_n(t) \tag{3}$$

where $s(t)$ is the signal from the astronomical source, and $N_n(t)$ is the noise. The signals from each telescope are added together to form a combined signal $y(t)$, given by:

$$y(t) = \sum_{n=1}^{N} \omega_n x_n(t) \tag{4}$$

where the ω_n terms are the complex weights applied to each antenna. These weights are crucial as they determine how much each antenna's signal contributes to the combined signal. The weights ω_n are calculated based on the desired beam direction θ and the positions of the antennas in the array. The ideal weights for a beamforming pattern are given by:

$$\omega_n = exp\left(\frac{-j2\pi d}{\lambda sin(\theta)}\right) \tag{5}$$

where d is the spacing between antennas, λ is the wavelength of the received signal, and θ is the desired direction of the beam. This equation ensures that the signals from different antennas combine constructively for the desired direction and cancel out for other directions.

The phase adjustment in beamforming is crucial for aligning the signals from different antennas. By adjusting the phase of each antenna's signal, they can be made to add up constructively in the desired direction and cancel out in other directions due to destructive interference. Beamforming allows for electronic steering of the beam without physically moving the antennas. By adjusting the weights ω_n appropriately, the combined beam can be steered towards different points in the sky. Once the combined beam is formed, it undergoes further signal processing such as filtering and demodulation to extract the astronomical information embedded in the signal. The processed data from beamforming is then analyzed to study various aspects of celestial objects, including their intensity, spectral characteristics, polarization, and spatial distribution.

3 Correlation, Visibility, and Sky Brightness

A correlator is a crucial component of an interferometer. It is a digital signal processing device that rapidly computes the cross-correlation function between pairs of antennas in the interferometer. The cross-correlation function is essentially a measure of how similar the signals are to one another. The correlator takes the time-series data from each antenna pair and calculates the correlation as a function of time delay or spatial separation. The cross-correlation function $C(u, v)$ for two signals $x_1(t)$ and $x_2(t)$ received by antennas separated by a baseline vector $\boldsymbol{b} = (b_x, b_y)$ is given by:

$$C(u, v) = \int x_1(t) x_2^*(t - \tau) e^{-2\pi i (u b_x + v b_y)} dt \tag{6}$$

where $u = \frac{b_x}{\lambda}$ and $v = \frac{b_y}{\lambda}$ are the spatial frequencies corresponding to the baseline vector $\boldsymbol{b}$, τ is the time delay between the signals, and λ is the wavelength.

The visibility function is derived from the cross-correlation function, and represents the spatial frequency components of the observed sky brightness distribution. It encapsulates information about the interference patterns created by the interferometer's baselines. The visibility function $V(u, v)$ is related to the cross-correlation function $C(u, v)$ through a Fourier transform:

$$V(u, v) = \iint T(x, y) e^{2\pi i (ux + vy)} dx dy \tag{7}$$

Here, $T(x, y)$ is the sky brightness distribution in the image plane, and x and l are the coordinates of that plane.

The sky brightness represents the intensity of radiation coming from celestial sources at different locations in the sky. In interferometric radio astronomy, the sky brightness distribution $T(x, y)$ is the target for imaging. It is related to the visibility function $V(u, v)$ through an inverse Fourier transform:

$$T(x, y) = \iint V(u, v)e^{-2\pi i(ux+vy)}dudv \tag{8}$$

This equation allows astronomers to reconstruct a radio image of the sky brightness distribution from interferometric data.

Figure 3: Radio image of the Hercules A galaxy taken by the Very Large Array (VLA) radio telescope. The jets of gas are caused by the supermassive black hole at the center of the galaxy.